LEVEL II

(12)

AD A099503

Variants of Uncertainty

Daniel Kahneman

University of British Columbia

Amos Tversky

Stanford University

May 15, 1981

Preparation of this report was supported by the
Engineering Psychology Programs, Office of Naval Research

ONR Contract N00014-79-C-0077 Work Unit NR 197-058

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
Technical Report No. 7

2. GOVT ACCESSION NO.
AD-A099503

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6 Variants of Uncertainty.

5. TYPE OF REPORT & PERIOD COVERED
9 Technical Report, Jan [redacted] 80 - Apr [redacted] 1981

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
10 Daniel Kahneman [redacted] Amos Tversky

8. CONTRACT OR GRANT NUMBER(s)
15 N00014-79-C-0077

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Stanford University
Department of Psychology, Building 420
Stanford, California 94305

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
NR 197-058

11. CONTROLLING OFFICE NAME AND ADDRESS
Engineering Psychology Programs
Office of Naval Research - Code 455
Arlington, Virginia 22217

12. REPORT DATE
11 15 May [redacted] 81

12 27

13. NUMBER OF PAGES
21

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
14 TR-7

15. SECURITY CLASS. (of this report)

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Perceptual expectancies
Ignorance
Surprise
Propensity

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In contrast to formal theories of judgment and decision, which employ a single notion of probability, psychological analyses of responses to uncertainty reveal a wide variety of processes and experiences, which may follow different rules. Elementary forms of expectation and surprise in perception are reviewed. A phenomenological analysis is described, which distinguishes external attributions of uncertainty (disposition) from internal attributions of uncertainty (ignorance). Assessments of uncertainty can be made in different modes, by fo-

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

Unclassified 403110

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block 20 continued:

cusing on frequencies, propensities, the strength of arguments, or direct experiences of confidence. These variants of uncertainty are associated with different expressions in natural language; they are also suggestive of competing philosophical interpretations of probability.

Accession For		
NTIS GRA&I		X
DTIC TAB		☐
Unannounced		☐
Justification		
By		
Distribution/		
Availability Codes		
Dist	Avail and/or Special	
A		

We are concerned in this paper with the multiplicity of states and experiences of uncertainty, and with the possibility that these states cannot all be described by a single concept. Analyses of uncertainty in philosophy, statistics and decision theory commonly treat all forms of uncertainty in terms of a single dimension of probability or degree of belief. Recent psychological studies of judgment under uncertainty have often followed this tradition and have focused on the correspondence of intuitive judgments to the standard logic of probability (Einhorn & Hogarth, 1981; Kahneman, Slovic, & Tversky, 1982; Kahneman & Tversky, 1981; Nisbett & Ross, 1980; Slovic, Fischhoff & Lichtenstein, 1977). A comprehensive psychological perspective on uncertainty, however, reveals a variety of processes and experiences, ranging from such basic mechanisms as habituation to repreated stimulation in a single neurone, to such complex activities as the evaluation of scientific hypotheses.

In this paper we sketch some extensions of the range of observations that are normally considered in psychological analyses of judgments under uncertainty. Two levels of responses to uncertainty are discussed. We first describe some basic processes of expectation and surprise in perception, which can be considered the precursors of subjective probability. We then turn to a phenomenological examination, in which we distinguish internal from external attributions of uncertainty and sketch four modes of judgment that people may adopt in assessing uncertainty.

ELEMENTARY FORMS OF PROBABILITY

Uncertainty is a fact with which all forms of life must be prepared to contend. At all levels of biological complexity there is uncertainty about the significance of signs or stimuli and about the possible conse-

quences of actions. At all levels, action must be taken before the uncertainty is resolved, and a proper balance must be achieved between a high level of specific readiness for the events that are most likely to occur and a general ability to respond appropriately when the unexpected happens. Because the focus of the present treatment is on belief rather than on action, we shall not discuss the remarkable processes by which lower organisms distribute their response effort in accordance with probabilities of reinforcement (Herrnstein, 1970). Our present concern in this section is mainly with perceptual uncertainty.

Perceptual Expectations

Before the event there are expectations. After the event there may be surprise. Surprise has been studied mainly by psychophysical methods, and it has been measured by the various indicators of the orienting response (Sokolov, 1969; Lynn, 1966) and by the P300 component of event-related potentials (Donchin, Ritter & McCallum, 1978; Duncan-Johnson & Donchin, 1977). Expectancies have been studied in many contexts, and by a wide variety of methods.

Our discussion of perceptual expectancies will be organized around the scheme shown in Figure 1, which distinguishes three main types of expectations. The first major distinction separates active from passive expectations: an active expectation occupies consciousness and draws on the limited capacity of attention; in contrast, a passive expectation is automatic and effortless, and is better described as a disposition than as an activity (Posner, 1978).

Insert Figure 1 here

Some expectancies are relatively permanent. Long-lasting expectancies about covariations of attributes define the perceptual categories that we use to organize and encode experience (Broadbent, 1971). Specific expectations about objects, e.g., that rooms and windows are likely to be rectangular, function as permanent assumptions which help determine the interpretation of ambiguous stimuli (Ittelson & Kilpatrick, 1951). We are chronically better prepared for some events than for others, as illustrated by the robust effect of past frequency on the recognition threshold for words (Broadbent, 1967; Morton, 1969). Indeed, expectations sometimes produce hallucinatory experiences that people cannot distinguish from real ones, as in the phonemic restoration effect. Thus, all the sensory information corresponding to the 's' in the word 'legislature' can be removed from a recording of the word, and be replaced by a cough, or by some other natural sound. Subjects who are exposed to this recording are utterly convinced that they heard the phantom phoneme (Warren, 1970).

Passive and temporary expectancies mediate the large effects of context on recognition (Foss & Blank, 1980), and several variants of priming effects (Posner, 1978). For example, the inclusion of a letter in the warning signal which introduces a trial facilitates the response to that letter in a speeded matching task, even when the contingencies are so arranged that the warning signal conveys no valid information about the target. Posner (1978) has documented some important differences between the passive expectation that is set up by an uninformative warning signal and the active expectation which is produced when the

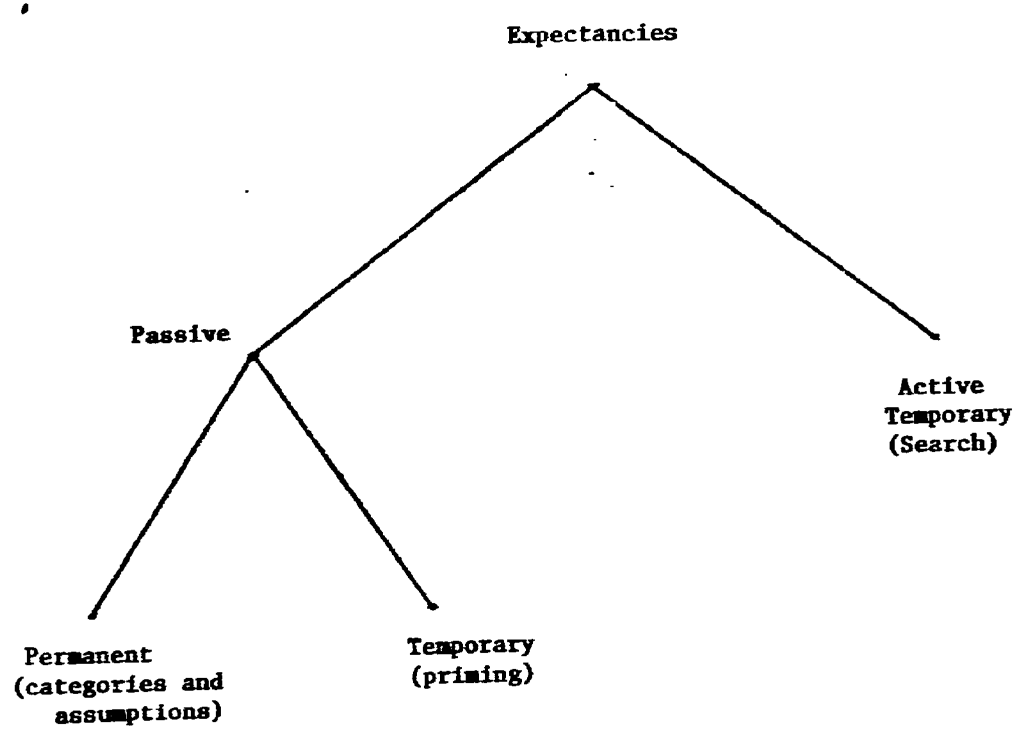

Figure 1. Perceptual expectancies.

target is in fact predictable, albeit imperfectly, from that signal. A passive expectation yields a benefit (i.e., a faster response) when it is confirmed, but it does not impede the response to targets that have not been primed. In contrast, a signal which causes the subject to prepare actively for a particular target also slows the response to unanticipated targets. In the language of probability theories, active expectations obey a principle of complementarity: a high degree of preparation for a particular event is achieved at the expense of a loss of preparation for other events. Passive priming is associated wth a non-complementary pattern of benefit without cost.

Passive expectations and conscious anticipations can conflict, and there is evidence that the passive process exerts greater influence on the interpretation of ambiguous stimuli. Epstein & Rock (1960) pitted the two types of expectations against one another, using a picture in which a left-looking and a right-looking profile were joined to form a pattern of reversible figure-ground organization. Observers of the composite picture only saw one of the profiles, which appropriated the common contour. Having constructed two profiles which could be joined in this fashion, Epstein & Rock presented the profiles separately in regular alternation for a number of trials, creating a conscious expectation that each would always be followed by the other. The composite was then presented for the first time, and the face that the subjects saw in it was recorded. In accord with the priming effect, the observers almost always saw the profile that had been shown on the preceding trial, rather than the one which they consciously expected to occur.

A related demonstration of a conflict between different levels of expectation has been reported, in which the P300 component of the EEG was the main dependent variable. The P300 is a positive deflection in the EEG, which occurs about 300 msec after the presentation of any

stimulus that the observer treats as significant or task-relevant. Many careful studies have demonstrated a close link between the prior probabilities of events and the magnitude of the P300 deflections that they elicit (Donchin, Ritter & McCallum, 1978). When a subject is exposed to a Bernoulli series, frequently repeated events elicit a smaller P300 than do rare ones. Furthermore, a run of repetitions of the same event is associated with a steadily decreasing P300, suggesting an increase in the subjective probability of further repetitions. In contrast, the conscious expectation of repetitions decreases consistently during a long run, by the familiar gambler's fallacy. Evidently, an observer can be prepared, or 'primed' for one event while consciously expecting another -- and can show physiological evidence of surprise at the occurrence of an event that was consciously predicted. Thus, there is a sense in which an individual can have conflicting probabilities for the same event at the same time. These observations suggest an image of the mind as a bureaucracy (Dennett, 1979) in which different parts have access to different data, assign them different weights and hold different views of the situation.

Perception as a Bet

Expectancies that have developed over a life-time of visual experience have a profound effect on perception, and are strikingly inaccessible to conscious knowledge or intention. The best-known demonstrations of these facts have been developed by the transactionalist students of perception (Ittelson & Kilpatrick, 1951; Kilpatrick, 1961). Observers of the famous distorted room and rotating window are led to have visual experiences that contradict both their general knowledge and their specific acquaintance with the objects of the illusions. Thus, one's

friends may be seen as giants or midgets, who change size as they walk along the wall of the distorted room, and a paper napkin may appear to slice through the rotating window. These striking effects are produced by the dominant assumption that rooms and windows are rectangular. Although the observer knows quite well that the assumption is not applicable to the case at hand, this knowledge has no significant effect on conscious perception. Models of reality that have been built over the years cannot be revised on demand for a particular occasion. These observations again confirm that an observer can simultaneously hold conflicting views of the same event.

We have noted that perceptual expectancies determine what we "see" in an ambiguous stimulus. Indeed, the transactionalists have interpreted perception as a bet on reality (Kilpatrick, 1961). A significant aspect of such perceptual choices is the strong commitment to the chosen interpretation. Our experience contains no indication of the equivocation of stimuli, and even when perceptual interpretations fluctuate over time, as with the Necker cube, they tend to be quite definite at any particular moment. The suppression of uncertainty and equivocation in perception suggests that we may be biologically programmed to act on the perceptual best bet, as if this bet involved no risk of error. A significant difference between the conscious experiences of perception and thought is that the latter can represent doubt and uncertainty, while the former normally do not.

Although the suppression of uncertainty distinguishes perceptual bets from conscious judgments about uncertain events, the processing of uncertainty at the two levels may be similar in other respects. Two striking observations of transactionalist research suggests hypotheses that seem to apply to conscious beliefs. The first is that the reconstructed image of the environment terds to be coherent, reflecting the

normal constraints and dependencies among the attributes of the scene and of the stimulus. Thus, when an object is presented under conditions that make both its size and its distance ambiguous, the chosen perceptual interpretation will select a size and a distance that relate to retinal size in the standard manner: if the object is seen as large, then it also appears to be further away than if it is seen small (Ittelson & Kilpatrick, 1951).

The second observarion is that perceptual construction appears to be a hierarchical process, in which decisions about the global features of the scene constrain and dominate decisions about the objects contained in it. The distorted room provides the best example. What is seen is not a compromise between two extreme views: normal-sized people in a distorted room, or oddly-sized people in a normal room. The latter view simply dominates the former, as if the shape of the room were computed before the processing of the people in it begins. Whether similar rules can be shown to operate, for example, in the construction of scenarios of future events is a problem that well deserves study.

THE PHENOMENOLOGY OF UNCERTAINTY

The preceding section sought to show that the rules that govern perceptual expectancies differ from the rules of probability theory. The present section extends this analysis to the experiences of doubt and uncertainty that judgments of subjective probability are assumed to reflect. As we shall see, the notion of probability refers in natural language to several distinct states of mind, to which the rules of the standard calculus of probability may not be equally applicable.

To appreciate the complexity of expectations, consider one of their manifestations: the surprise that we experience when an expectation is

violated. Imagine that a coin is to be tossed 40 times. What number of "heads" would you expect? If you assume that the coin is fair, you would probably state that the 20-20 result is more likely than any other, yet you would be more surprised by this outcome than by a result of 22 "heads" and 18 "tails". Is the 'true' subjective probability of the two events indicated by the considered judgment of their relative likelihood, or by the involuntary reaction of surprise which they would elicit?

One possible interpretation is that the example illustrates a conflict between two approaches to the judgment of probability: the judgment that the most likely outcome is 20-20 derives from knowledge of the rules of chances, but outcomes such as 22-18 or 17-23 are more probable at another level where probability is determined by representativeness. A slightly uneven outcome represents both the fairness of the coin and the randomness of tossing, which is not at all represented by the exactly even result. In this view, the greater psychological reality of expectations based on representativeness manifests itself in the surprise reaction.

A slightly different interpretation is possible, which focuses on the coding of the possible outcomes. As we shall see, it is frequently appropriate in conversation to extend the definition of an event X to "X or something like it". If the spontaneous coding of events follows similar rules, outcomes such as 22-18 or 17-23 will be spontaneously coded as "an approximately even split", while the outcome 20-20 will be assigned a distinctive code of "exactly even split". A person who attempts to judge the relative likelihood of the events will consider the explicit statement of the outcomes, and note that 20-20 is more likely than, say, 22-18. But the reaction of surprise may be determined by the natural coding of events. The event 22-18 will then be relatively un-

surprising because it is coded as an approximately even result, which is indeed more likely than a precisely even one.

The role of event-coding is manifest in the interpretation of uncertain assertions, such as "I estimate that . . .", or, sometimes "I think that . . .". Uncertain assertions are a class of speech acts, which are characterized by specific sincerity conditions and tests of validity. Consider, for example, the prediction: "I think that the price of gold will be higher by 50% in six months than it is today". Taken literally, this is a point-prediction, which should be assigned a very small probability of confirmation. But the prediction is not intended to be taken literally. Point predictions are normally understood as comparative statements, or as statements of the range in which an outcome is expected to fall, e.g., "I think the increase in the price of gold will be nearer to 50% than to X% or Y%". The speaker and the listener normally expect to agree on the tacitly implied values of X and Y. For example, the forecaster cited above will be considered remarkably accurate if the price of gold actually rises by 53% in the next six months, although the forecast was not strictly true. Thus, a speaker who asserts a numerical prediction is committed to a range rather than to a point. The speaker is also committed to the proposition that the value is about equally likely to be above the estimate as below it, except when the nature of the prediction makes this impossible. Thus, a person who says "I think the price of gold will rise by 50% in the next six months" would be considered to be deliberately misleading if he or she also thought, but did not communicate, that the actual value was much more likely to be above the estimate than below it.

It is significant that the sincerity conditions associated with a prediction do not require that the predicted value (or range) of a variable be considered highly probable, but only that it be considered more

probable than comparable values (or ranges). For example, a man who asserts "I think Billy John will win the gold medal for the high jump in the next Olympics" will not be considered a liar if he prefers to bet against this proposition rather than on it, but he is prohibited from adding: "and the chances of Jack Small are evon better". Thus, natural language allows a privileged role to the best guess, and the identification of the favored guess conveys information about the alternatives to which it may fairly be compared. The mention of a particular favorite athlete indicates that he is to be compared to other individual athletes, rather than to a disjunction of possible winners. One consequence of this rule is that it is sometimes possible to "predict" an event which is considered less probable than its complement, if the complement is naturally coded as a disjunction.

A related restriction applies to expressions of confidence. A statement of confidence expresses one's uncertainty in a prediction, estimate or inference to which one is already committed. Thus, it is natural to ask "how confident are you that you are right?", but it is anomalous to ask: "how confident are you that you are wrong?". Confidence is the subjective probability or degree of belief associated with what we "think" will happen.

Common language also provides a large number of expressions to talk of events which may happen, although we do not necessarily "think" they will. Thus, people assess the chances of candidates, estimate the risks of different activities, give odds for football games and understand forecasters' statements about the probability of rain. We now turn to a more detailed analysis of the states of uncertainty which such statements may express, following the scheme shown in Figure 2. The two levels of the figure, attributions of uncertainty and variants of uncertainty, are discussed in the following sections.

Insert Figure 2 here

Attributions of Uncertainty

The primary distinction shown in Figure 2 refers to two loci to which uncertainty can be attributed: the external world or our state of knowledge. For example, we attribute to causal systems in the real world the uncertainty associated with the tossing of a coin, the drawing of a hand of cards from a pack, the outcome of a football game and the behavior of the St. Helens volcano. These causal systems have dispositions to produce different events, and we judge the probabilities of these events by assessing the relative strength of the competing dispositions. In contrast, such statements as "I think Mt. Blanc is the tallest mountain in Europe" or "I hope I spelled her name correctly" reflect an uncertainty that is attributed to one's mind rather than to a mountain or a woman. (Howell and Burnett (1978) have applied the terms internal and external uncertainty, respectively, to events that the subjects can or cannot control.)

Our distinction between ignorance and external uncertainty is closely related to a more general distinction between internal and external attributions of experience. Color, size and texture, for example, are normally experienced as properties that belong to external objects, but pains, feelings and memories are attributed to the experiencing subject rather than to the eliciting object.

The attribution of uncertainty can sometimes be inferred from a simple linguistic test: is it appropriate to describe the assessment of uncertainty as "the probability is . . ."? Or should one say "my proba-

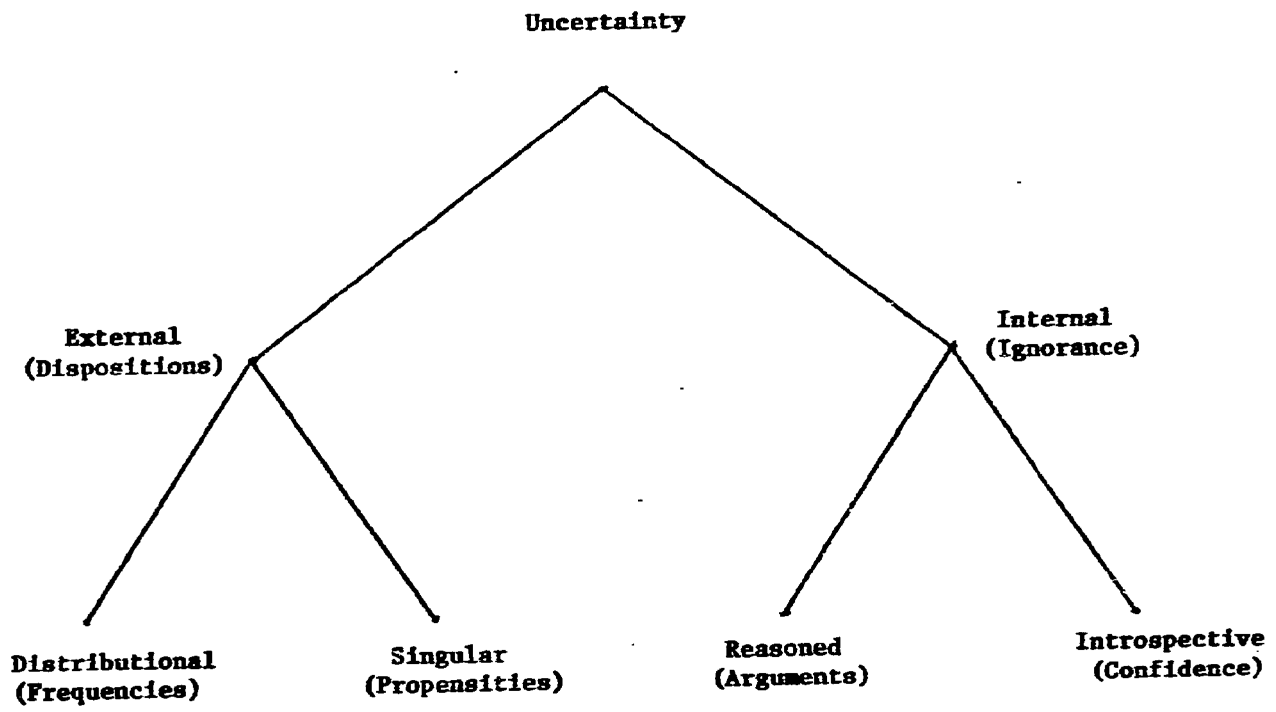

Figure 2. Variants of uncertainty.

bility is . . ."? In contrast to the Bayesian view, which treats all probabilities as subjective and personal, natural language marks the distinction between internal and external uncertainty. Thus it is legitimate to speak of "the best estimate of the probability of a change of regime in Saudi Arabia within the next year", but it is anomalous to say "the best estimate of the probability that the Nile is the largest river in the world is . . .". Best estimates of probability belong to the public domain. Expressions of private ignorance do not.

This test does not always distinguish internal from external uncertainty. For example, one may speak of <u>the</u> probability that Marlowe wrote Hamlet although this uncertainty is attributed to our ignorance rather than to the strength of Marlowe's propensity to write plays. The use of "<u>the</u> probability" in this example is justified by the existence of a public body of knowledge, which reduces but does not eliminate the uncertainty about the authorship of Hamlet. Not everybody need have access to this knowledge, but the estimated probability refers to a reasonable or consensual inference from the available evidence. In the example of the Nile, however, the public body of evidence certainly includes the correct answer, and ignorance can only be private.

The attribution of uncertainty about an event to dispositions or to ignorance depends, among other things, on timing. Uncertainty about past events is likely to be experienced as ignorance, especially if the truth is known to someone else, whereas uncertainty about the future is more naturally attributed to the dispositions of the relevant system. Indeed, it has been noted that people exhibit different attitudes to the outcome of a coin toss, depending on whether or not the coin has already been tossed (Rothbart & Snyder, 1970).

Variants of Uncertainty

The second level of Figure 2 distinguishes four prototypical variants of uncertainty, identified by the nature of the data that the judge might consider in evaluating probability. External uncertainty can be assessed in two ways: (i) a distributional mode, where the case in question is seen as an instance of a class of similar cases, for which the relative frequencies of outcomes are known, or can be estimated; (ii) a singular mode, in which probabilities are assessed by the propensities of the particular case at hand. The two modes of judgment are illustrated by the following true story.

A team that was concerned with the development of a high school curriculum on thinking under uncertainty was conducting a planning session. The question was raised of the time that would be required to complete the first version of a textbook. The participants in the discussion were asked to estimate this value as realistically as possible; the seven estimates ranged from 18 months to three years. The team leader then turned to one of the participants, an educator wth considerable expertise in the problems of curriculum development, with the following question: "What has been the experience of other teams that have tried to write a textbook and develop a curriculum in a new area, where no previous course of study existed? How long did it take them to complete a textbook, from a stage comparable to the present state of our project?". The chilling implications of the answer appeared to surprise the expert who gave it, much as they surprised the other participants: "Most teams I could think of failed, and never completed a textbook. For those that succeeded, completion times have ranged from five to nine years, with a median of seven."

Subsequent probing revealed that all participants had produced

their initial estimate in the singular mode, by constructing plans and scenarios, with some allowance of safety margins for unforeseen contingencies. Because of anchoring effects (Tversky & Kahneman, 1974) an estimate which is obtained by adding safety margins to current plans is likely to be highly optimistic. A notable aspect of this anecdote is that the relevant distributional information was not spontaneously used, although it was available to one expert from personal knowledge, and could have been estimated quite accurately by several other participants.

Our example illustrated the application of singular and distributional modes of reasoning to the prediction of a continuous variable: the time required to complete a project. The distributional information consisted in this case of knowledge about the relative frequencies of different completion times. Of course, a similar reasoning can be applied to assess the probability of a discrete outcome, such as the failure of the project. The relative frequency of that outcome in a relevant class provides the basis for a distributional assessment of probability, and other information about the particular case, used in the singular mode, may produce an impression of propensity to fail or to succeed. There are many instances in which the same question can be approached in either singular or distributional mode.

Compare the following examples:

(1) "Chances are that you will find John at home if you call tomorrow morning. He said that he prefers to work at home."

(2) "Chances are that you will find John at home if you call tommorrow morning. He has often been there when I called him."

Statement 1 only allows a singular judgment of the probability that John will be at home. Statement 2 could support both a distributional and a singular assessment. The relative frequency of similar mornings

on which John has been at home provides a natural estimate of the probability of finding him there tomorrow, but the statment has also endowed John with a propensity to spend mornings at home, much as did Statement 1.

We have conjectured (Kahneman & Tversky, 1979) that people generally prefer the singular mode, in which they take an "inside view" of the causal system that most immediately produces the outcome, over an "outside view", which relates the case at hand to a sampling schema. Our planning example illustrates this preference for the singular mode. It also illustrates another effect, which we suspect to be quite general: that the distributional mode of judgment is more likely than the singular to yield accurate estimates of values and reasonable assessments of probability.

We now turn to a distinction between two modes of assessment of internal uncertainty, which are illustrated by the following examples:

(3) "I believe New York is north of Rome, but I am not sure."

(4) "I think her name is Doris, but I am not sure."

The uncertainty expressed in these statements is clearly internal: the statements reflect (partial) ignorance rather than dispositions of external objects. It is surely far-fetched to speak of the propensity of New York to be north of Rome (incidentally, it is not), or of Linda to be remembered as Doris.

The two statements differ in the nature of the evidence on which they are based. Statement 3 could reflect a process of sifting and weighing of evidence and arguments (e.g., New York is much colder than Rome; Rome is in the middle of Italy, etc.). Statement 4 has a different character. The confidence that it expresses is based on an introspective judgment of the strength of an association. Much as happens when we check the spelling of a word by examining whether it "looks

right", confidence rests on an unanalyzed experience. In studies of psychophysics and of memory, the confidence associated with judgments is significantly correlated with accuracy: people are more likely to be confident when they are correct than when they are not, although their assessments of the probability that they are right are poorly calibrated (see Lichtenstein, Fischhoff & Phillips, 1982).

As in the case of external uncertainty, the internal uncertainty associated with a given question can sometimes be assessed both in the reasoned and in the introspective modes. For example, a question concerning the age of a movie star can be approached introspectively by searching for an answer that sounds familiar, or in a reasoned mode by trying to induce the answer from other knowledge.

We do not wish to suggest that any experience of uncertainty can be assigned to one of the four variants of Figure 2. There are undoubtably many mixed and indeterminate cases. We have seen that the uncertainty in a given problem can be attributed to external dispositions, to one's ignorance, or to a combination of the two, and that it may be assessed in a singular mode, in a distributional mode, or in a mixture of modes. The purpose of our treatment was to highlight some significant dimensions of variation in experiences of uncertainty, not to offer an exhaustive and mutually exclusive classification of these experiences. For an attempt to classify experimental operations in the measurement of subjective probability, see Howell and Burnett (1978).

Discussion

Although the language of probability can be used to express any form of uncertainty, the laws of probability theory do not apply to all variants of uncertainty with equal force. These laws are most likely to

be accepted, and satisfied in intuitive judgments, when an external uncertainty is assessed in a distributional or frequentistic mode. For example, complementarity of subjective probability is very compelling when we consult weather statistics in order to assess the probability that it will rain next year on April 12: the relevant set of past April days is clearly separable into days on which there was rain and days on which there was not.

Complementarity is less compelling in other variants. When uncertainty is assessed in terms of propensities, arguments or confidence, it is less obvious that the probabilities should add up to unity -- even if it is known with certainty that one of the alternatives is correct. For example, one may question why the degree of belief in the assertion that New York is north of Rome and the degree of belief in the assertion that New York is south of Rome should sum to the same value as the degrees of belief for any other pair of complementary statements. Indeed, several authors (e.g., Cohen, 1977, Shafer, 1976) have proposed that complementarity should not apply to degree of belief. In particular, Shafer has argued against complementarity of belief on the grounds that there are situations in which two mutually exclusive and exhaustive hypotheses both have substantial support, and other situations in which neither hypothesis has much support. Similar questions could be raised about the necessity of complementarity in impressions of confidence, and in assessments of conflicting propensities.

The variants of uncertainty may differ in the confidence with which they are assessed. Imagine that a thumb tack has been tossed four times, and has landed twice on its point and twice on its head. Given these data, most observers will assign a probability of .5 to the event that the thumb tack will land on its head on the next toss. They also assign a probability of .5 to the event that a tossed coin will show

"heads", but express much greater confidence in their judgment about the coin than about the tack. As this example illustrates, it is quite possible to assign different degrees of confidence to the same judgment of propensity. Confidence about probabilities is important because it controls decisions. There is evidence (Ellsberg, 1961; Raiffa, 1961) that people prefer to bet on events that have known probabilities, such as the toss of a coin, rather than on events that are associated with a combination of external uncertainty and ignorance, such as the toss of a thumb tack.

There are natural links between the conceptions of probability advanced by different schools of thought on this topic and the modes of uncertainty that we have discussed. Thus, the frequentistic or objective interpretation of probability restricts the concept to external uncertainty generated by a sampling process. In contrast, the Bayesian or personal school treats all uncertainty as ignorance. In the Bayesian school, preferences are the basis of beliefs, and probabilities are derived from preferences between bets. From a psychological point of view, however, this betting heuristic appears unrealistic. Controversy has often been sharp in this domain, because of the existence of intuitions which are individually compelling and mutually incompatible, and because there is no agreed criterion for settling normative disputes when intuitions conflict. A psychological analysis could perhaps contribute to the normative discussion, by providing an adequate description of the intuitions from which the various positions draw their appeal.

References

Broadbent, D.E. Word-frequency effect and response bias. Psychological Review, 1967, 74, 1-15.

Broadbent, D.E. Decision and stress. London: Academic Press, 1971.

Cohen, L.J. The probable and the provable. Oxford: Claredon Press, 1977.

Dennett, D.C. Brainstorms. Hassocks: Harvester, 1979.

Donchin, E., Ritter, W. & McCallum, W.C. Cognitive psychophysiology: The endogeneous components of the ERP. In E. Callaway, P. Tueting & S.H. Koslow (Eds.), Event-related brain potentials in man. New York: Academic Press, 1978.

Duncan-Johnson, C.C. & Donchin, E. On quantifying surprise: The variation of event-related potentials with subjective probability. Psychophysiology, 1977, 14, 456-467.

Einhorn, H.J. & Hogarth, R.M. Behavioral decision theory: Processes of judgment and choice. Annual Review of Psychology, 1981, 32, 53-88.

Ellsberg, D. Risk, ambiguity and the Savage axioms. Quarterly Journal of Economics, 1961, 75, 643-699.

Epstein, W. & Rock, I. Perceptual set as an artifact of recency. American Journal of Psychology, 1960, 73, 214-228.

Foss, D.J. & Blank, M.A. Identifying the speech codes. Cognitive

Psychology, 1980, *12*, 1-31.

Herrnstein, R.J. On the law of effect. *Journal of the Experimental Analysis of Behavior*, 1970, *13*, 243-266.

Howell, W.C. & Burnett, S.A. Uncertainty measurement: A cognitive taxonomy. *Organizational Behavior and Human Performance*, 1978, *22*, 45-68.

Ittelson, W.H. & Kilpatrick, F.P. Experiments in perception. *Scientific American*, 1951, *185*, 50-55.

Kahneman, D., Slovic, P. & Tversky, A. (Eds.). *Judgment under uncertainty: Heuristics and biases*. New York: Cambridge University Press, 1982.

Kahneman, D. & Tversky, A. Intuitive prediction: Biases and corrective procedures. *TIMS Studies in Management Science*, 1979, *12*, 313-327.

Kahneman, D. & Tversky, A. On the study of statistical intuitions. *Cognition*, 1981, in press.

Kilpatrick, F.P. *Explorations in transactional psychology*. New York: New York University Press, 1961.

Lichtenstein, S., Fischhoff, B. & Phillips, L. Calibration of probabilities: The state of the art to 1980. In D. Kahneman, P. Slovic & A. Tversky (Eds.), *Judgment under uncertainty: Heuristics and biases*. New York: Cambridge University Press, 1982.

Lynn, R. *Attention, arousal and the orientation reaction*. Oxford: Pergamon, 1966.

Morton, J. Interaction of information in word recognition. Psychological Review, 1969, 76, 165-178.

Nisbett, R.E. & Ross, L. Human inference: Strategies and shortcomings. Englewood Cliffs, N.J.: Prentice-Hall, 1980.

Posner, M.I. Chronometric explorations of mind. Hillsdale: Lawrence Earlbaum Associates, 1978.

Raiffa, M. Risk, ambiguity and the Savage axioms: Comment. Quarterly Journal of Economics, 1961, 75, 690-694.

Rothbart, M. & Snyder, M. Confidence in the prediction and postdiction of an uncertain event. Canadian Journal of Behavioral Science, 1970, 2, 38-43.

Shafer, G. A mathematical theory of evidence. Princeton: Princeton University Press, 1976.

Slovic, P., Fischhoff, B. & Lichtenstein, S. Behavioral decision theory. Annula Review of Psychology, 1977, 28, 1-39.

Sokolov, E.N. The modeling properties of the nervous system. In I. Maltzman & K. Cole (Eds.), Handbook of comtemporary Soviet psychology. New York: Basic Books, 1969.

Tversky, A. & Kahneman, D. Judgment under uncertainty: Heuristics and biases. Science, 1974, 185, 1124-1131.

Warren, R.M. Perceptual restoration of missing speech sounds. Science, 1970, 167, 393-395.

Distribution List

OSD

CDR Paul R. Chatelier
Office of the Deputy Under Secretary
of Defense
OUSDRE (E&LS)
Pentagon, Room 3D129
Washington, D.C. 20301

Department of the Navy

Director
Engineering Psychology Programs
Code 455
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217 (5 cys)

Director
Communication & Computer Technology
Code 240
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

Director
Manpower, Personnel & Training
Code 270
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

Director
Operations Research Programs
Code 434
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

Director
Statistics and Probability Program
Code 436
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

Director
Information Systems Program
Code 437
800 North Quincy Street
Arlington, VA 22217

Department of the Navy

Code 430B
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

Special Assistant for Marine
Corps Matters
Code 100M
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

Commanding Officer
ONR Eastern/Central Regional Office
ATTN: Dr. J. Lester
Building 114, Section D
666 Summer Street
Boston, MA 02210

Commanding Officer
ONR Branch Office
ATTN: Dr. C. Davis
536 South Clark Street
Chicago, IL 60605

Commanding Officer
ONR Western Regional Office
ATTN: Dr. E. Gloye
1030 East Green Street
Pasadena, CA 91106

Office of Naval Research
Scientific Liaison Group
American Embassy, Room A-407
APO San Francisco, CA 96503

Director
Naval Research Laboratory
Technical Information Division
Code 2627
Washington, D.C. 20375 (6 cys)

Dr. Robert G. Smith
Office of the Chief of Naval
Operations, OP987H
Personnel Logistics Plans
Washington, D.C. 20350

Department of the Navy

Naval Training Equipment Center
ATTN: Technical Library
Orlando, FL 32813

Human Factors Department
Code N215
Naval Training Equipment Center
Orlando, FL 32813

Dr. Alfred F. Smode
Training Analysis and Evaluation Group
Naval Training Equipment Center
Code N-00T
Orlando, FL 32813

Dr. Gary Poock
Operations Research Department
Naval Postgraduate School
Monterey, CA 93940

Dean of Research Administration
Naval Postgraduate School
Monterey, CA 93940

Mr. Warren Lewis
Human Engineering Branch
Code 8231
Naval Ocean Systems Center
San Diego, CA 92152

Dr. A.L. Slafkosky
Scientific Advisor
Commandant of the Marine Corps
Code RD-1
Washington, D.C. 20380

Mr. Arnold Rubinstein
Naval Material Command
NAVMAT 0722 - Rm. 508
800 North Quincy Street
Arlington, VA 22217

Commander
Naval Air Systems Command
Human Factor Programs
NAVAIR 340F
Washington, D.C. 20361

Mr. Phillip Andrews
Naval Sea Systems Command
NAVSEA 0341
Washington, D.C. 20362

Department of the Navy

Commander
Naval Electronics Systems Command
Human Factors Engineering Branch
Code 4701
Washington, D.C. 20360

Dr. Arthur Bachrach
Behavioral Sciences Department
Naval Medical Research Institute
Bethesda, MD 20014

CDR Thomas Berghage
Naval Health Research Center
San Diego, CA 92152

Dr. George Moeller
Human Factors Engineering Branch
Submarine Medical Research Lab
Naval Submarine Base
Groton, CT 06340

Commanding Officer
Naval Health Research Center
San Diego, CA 92152

Dr. James McGrath, Code 302
Navy Personnel Research and Development Center
San Diego, CA 92152

Navy Personnel Research and Development Center
Planning and Appraisal
Code 04
San Diego, CA 92152

Navy Personnel Research and Development Center
Management Systems, Code 303
San Diego, CA 92152

Navy Personnel Research and Development Center
Performance Measurement & Enhancement
Code 309
San Diego, CA 92152

Mr. Ronald A. Erickson
Human Factors Branch
Code 3194
Naval Weapons Center
China Lake, CA 93555

Department of the Navy

Dean of Academic Departments
U.S. Naval Academy
Annapolis, MD 21402

LCDR W. Moroney
Code 55MP
Naval Postgraduate School
Monterey, CA 93940

Mr. Merlin Malehorn
Office of the Chief of Naval
Operations (OP-115)
Washington, D.C. 20350

Dr. Carl E. Englund
Environmental Physiology Department
Ergonomics Program, Code 8060
Naval Health Research Center
P.O. Box 85122
San Diego, CA 92138

Department of the Army

Mr. J. Barber
HQS, Department of the Army
DAPE-MBR
Washington, D.C. 20310

Dr. Joseph Zeidner
Technical Director
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Director, Organizations and
Systems Research Laboratory
U.S. Army Research Insitute
5001 Eisenhower Avenue
Alexandria, VA 22333

Technical Director
U.S. Army Human Engineering Labs
Aberdeen Proving Ground, MD 21005

MAJ. Gerald P. Kreuger
USA Medical R&D Command HQ
SGRD-PLC
Fort Detrick, MD 21801

ARI Field Unit-USAREUR
ATTN: Library
C/O ODCSPER
HQ USAREUR & 7th Army
APO New York 09403

Department of the Air Force

U.S. Air Force Office of Scientific
Research
Life Sciences Directorate, NL
Bolling Air Force Base
Washington, D.C. 20332

Dr. Donald A. Topmiller
Chief, Systems Engineering Branch
Human Engineering Division
USAF AMRL/HES
Wright-Patterson AFB, OH 45433

Air University Library
Maxwell Air Force Base, AL 36112

Dr. Gordon Eckstrand
AFHRL/ASM
Wright-Patterson AFB, OH 45433

Dr. Earl Alluisi
Chief Scientist
AFHRL/CCN
Brooks AFB, TX 78235

Foreign Addresses

North East London Polytechnic
The Charles Myers Library
Livingstone Road
Stratford
London E15 2LJ
ENGLAND

Professor Dr. Carl Graf Hoyos
Institute for Psychology
Technical University
8000 Munich
Arcisstr 21
FEDERAL REPUBLIC OF GERMANY

Dr. Kenneth Gardner
Applied Psychology Unit
Admiralty Marine Technology
Establishment
Teddington, Middlesex TW11 OLN
ENGLAND

Director, Human Factors Wing
Defense & Civil Institute of
Environmental Medicine
Post Office Box 2000
Downsview, Ontario M3M 3B9
CANADA

Foreign Addresses

Dr. A.D. Baddeley
Director, Applied Psychology Unit
Medical Research Council
15 Chaucer Road
Cambridge, CB2 2EF
ENGLAND

Other Government Agencies

Defense Technical Information Center
Cameron Station, Bldg. 5
Alexandria, VA 22314 (12 cys)

Dr. Craig Fields
Director, Cybernetics Technology Office
Defense Advanced Research Projects Agency
1400 Wilson Blvd.
Arlington, VA 22209

Dr. Judith Daly
Cybernetics Technology Office
Defense Advanced Research Projects Agency
1400 Wilson Blvd
Arlington, VA 22209

Professor Douglas E. Hunter
Defense Intelligence School
Washington, D.C. 20374

Other Organizations

Dr. Robert R. Mackie
Human Factors Research, Inc.
5775 Dawson Avenue
Goleta, CA 93017

Dr. Gary McClelland
Institute of Behavioral Sciences
Unviversity of Colorado
Boulder, CO 80309

Dr. Miley Merkhofer
Stanford Research Institute
Decision Analysis Group
Menlo Park, CA 94025

Dr. Jesse Orlansky
Institute for Defense Analyses
400 Army-Navy Drive
Arlington, VA 22202

Other Organizations

Professor Judea Pearl
Engineering Systems Department
University of California-Los Angeles
405 Hilgard Avenue
Los Angeles, California 90024

Professor Howard Raiffa
Graduate School of Business Administration
Harvard University
Soldiers Field Road
Boston, MA 02163

Dr. T.B. Sheridan
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139

Dr. Arthur I. Siegel
Applied Psychological Services, Inc.
404 East Lancaster Street
Wayne, PA 19087

Dr. Paul Slovic
Decision Research
1201 Oak Street
Eugene, Oregon 97401

Dr. Robert T. Hennessy
NAS - National Research Council
JH #819
2101 Constitution Ave., N.W.
Washinton, D.C. 20418

Dr. M.G. Samet
Perceptronics, Inc.
6271 Variel Avenue
Woodland Hills, CA 91364

Dr. Robert Williges
Human Factors Laboratory
Virginia Polytechnical Institute and State University
130 Whittemore Hall
Blacksburg, VA 24061

Dr. Alphonse Chapanis
Department of Psychology
The Johns Hopkins University
Charles and 34th Streets
Baltimore, MD 21218

Other Organizations

Dr. Meredith P. Crawford
American Psychological Association
Office of Educational Affairs
1200 17th Street, NW
Washington, D.C. 20036

Dr. Ward Edwards
Director, Social Science Research Institute
University of Southern California
Los Angeles, CA 90007

Dr. Charles Gettys
Department of Psychology
University of Oklahoma
455 West Lindsey
Norman, OK 73069

Dr. Kenneth Hammond
Institute of Behavioral Science
University of Colorado
Room 201
Boulder, CO 80309

Dr. Willima Howell
Department of Psychology
Rice University
Houston, TX 77001

Journal Supplement Abstract Service
American Psychological Association
1200 17th Street, NW
Washington, D.C. 20036 (3 cys)

Dr. Richard W. Pew
Information Sciences Division
Bolt Beranek & Newman, Inc.
50 Moulton Street
Cambridge, MA 02138

Dr. Hillel Einhorn
University of Chicago
Graduate School of Business
1101 E. 58th Street
Chicago, IL 60637

Mr. Tim Gilbert
The MITRE Corporation
1820 Dolly Madison Blvd
McLean, VA 22102

Dr. Douglas Towne
University of Southern California
Behavioral Technology Laboratory
3716 S. Hope Street
Los Angeles, CA 90007

Other Organizations

Dr. John Payne
Duke University
Graduate School of Business Administration
Durham, NC 27706

Dr. Baruch Fischhoff
Decision Research
1201 Oak Street
Eugene, Oregon 97401

Dr. Andrew P. Sage
University of Virginia
School of Engineering and Applied Science
Charlottesville, VA 22901

Dr. Leonard Adelman
Decisions and Designs, Inc.
8400 Westpark Drive, Suite 600
P.O. Box 907
McLean, VA 22101

Dr. Lola Lopes
Department of Psychology
University of Wisconsin
Madison, WI 53706

Mr. Joseph Wohl
The MITRE Corp.
P.O. Box 208
Bedford, MA 01730